On the disk arretion onto a relativistic radiating star

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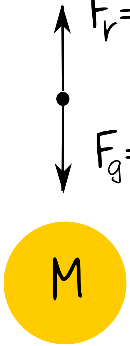
24.5.2018, $H\rho\alpha'\kappa\lambda\varepsilon\iota o$

Eddington luminosity



$$F_r = \frac{\sigma}{c}\frac{L}{4\pi r^2}$$

$$F_g = \frac{GmM}{r^2}$$

$$L_{Edd} = \frac{4\pi c G m M}{\sigma}$$

- Radiation changes the dynamics

  → Momentum transfer

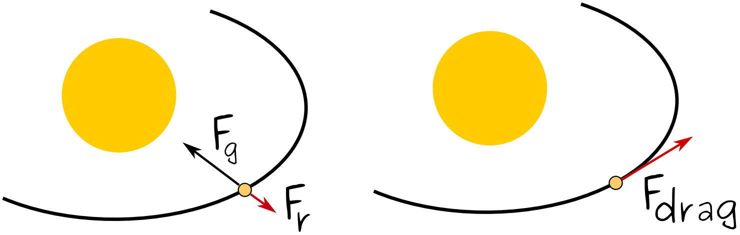
- Many (neutron) stars accrete close to Eddington limit.

  → How does the momentum transfer changes the disk accretion?

Three effects of irradiation

"Never start to solve the problem before you know the answer"

1. The irradiation changes the local thermal balance in the disk
   ⇒ Different energetics.
2. The radiation force changes the radial momentum balance
   ⇒ Different rotation profile.
3. The radiation changes the angular momentum balance
   ⇒ Different viscous heating.



Constructing the model (ingrediences)

- Spacetime: Static axially symmetric spacetime metric

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\varphi\varphi}d\varphi^2 + dz^2$$

- Fluid: Stress-energy tensor including (α-)shear viscosity:

$$T^\alpha_\beta = \left(e_{\rm gas} + p_{\rm gas}\right) u^\alpha u_\beta + p_{\rm gas}\delta^\alpha_\beta + w^\alpha_\beta$$

- Dynamics: Relativistic conservation laws

$$\nabla_\alpha \left(\rho u^\alpha\right) = 0$$
$$\nabla_\alpha T^\alpha_\beta = \phi_\beta$$

- Simplified radiative transfer:
  → grey material, isotropic emissivity, opacity and scattering
  → Radiation force ϕ_β evaluated in the fluid comoving frame
- Vertical integration
- no advection

Dynamical equations (Standard with RHSs)

Mass conservation:

$$(-g)^{1/2}\, g_{rr}^{-1/2} \gamma_r \Sigma V = -\frac{\dot{M}}{2\pi}$$

Angular momentum conservation:

$$-\frac{\dot{M}}{2\pi}\frac{d\ell}{dr} + \frac{d}{dr}\left(\sqrt{-g}W^r_\varphi\right) = \sqrt{-g}\Phi_\varphi$$

Radial momentum conservation:

$$\Sigma\left[\gamma_r^2 V \frac{dV}{dr} + \frac{g_{\varphi\varphi,r}}{2g_{tt}}\gamma_\varphi^2\left(\Omega^2 - \Omega_{\mathrm{K}}^2\right)\right] + \frac{dP_{\mathrm{gas}}}{dr} = \frac{1}{\gamma_r^2} h_r^\beta \Phi_\beta$$

Vertical momentum conservation:

$$\Sigma\Omega_\perp^2 z + \frac{\partial p_{\mathrm{gas}}}{\partial z} = \phi_z$$

Energy conservation:

$$q^+_{\mathrm{vis}} = q^-_{\mathrm{rad}}$$

Simplified radiative transfer

We split the radiation field into two components:

$$I(\tau, n^{\hat{\alpha}}) = I_1(\tau, n^{\hat{\alpha}}) + I_2(\tau, n^{\hat{\alpha}})$$

- $I_1(\tau, n^{\hat{\alpha}})$: Incident radiation from the star *(highly anisotropic)*
- $I_2(\tau, n^{\hat{\alpha}})$: Reprocessed and thermal radiation *(nearly isotropic)*

⇒ Two radiative-transfer equations:

$$\mu \frac{dI_1}{d\tau} = I_1,$$
$$\mu \frac{dI_2}{d\tau} = I_2 - J_1 - J_2 - q.$$

$I_1(\tau, n^{\hat{\alpha}}) \rightarrow$ formal solution, $I_2(\tau, n^{\hat{\alpha}}) \rightarrow$ Eddington approximation.

$$\frac{dH_2}{d\tau} = -J_1 - q, \quad \frac{dJ_2}{d\tau} = 3H_2, \quad q \equiv \frac{q^{-}_{\rm rad}}{4\pi\rho(\kappa^{\rm a} + \kappa^{\rm s})}$$

...Hansen (2008)

Solution of the RTEs

1. Incident radiation:

$$I_1\left(\tau, n^{\hat{z}}\right) = \begin{cases} I_{\rm irr}\left(n^{\hat{z}}\right) e^{\tau/n^{\hat{z}}} & (n^{\hat{z}} < 0) \\ I_{\rm irr}\left(-n^{\hat{z}}\right) e^{-(2\tau_h - \tau)/n^{\hat{z}}} & (n^{\hat{z}} > 0) \end{cases}$$

2. Reprocessed and thermal radiation:

$$H_2(\tau) = -\frac{1}{4\pi}\int_{n^{\hat{z}}<0}\left[e^{\tau/n^{\hat{z}}} + e^{(2\tau_h-\tau)/n^{\hat{z}}}\right] I_{\rm irr}(n)\, d\Omega + \int_{\tau}^{\tau_h} q(t)dt$$

At the surface of the disk ($\tau = 0$) this can be written as

$$\boxed{F_2(0) = 4\pi H_2(0) = -T_{\rm irr}^{\hat{t}\hat{z}} + \frac{1}{2}Q_{\rm rad}^{-},}$$

The expression for $H_2(\tau)$ is too complicated

Radiation force

In general it is given by

$$\phi^{\hat{\alpha}} = \oint \rho \left[\left(\kappa^{\mathrm{a}} + \kappa^{\mathrm{s}}\right) I - \frac{j}{4\pi} - \kappa^{\mathrm{s}} J \right] n^{\hat{\alpha}} d\Omega.$$

Using the RTE solutions

$$\Phi^{\hat{t}} = -Q^{-}_{\mathrm{rad}}$$

$$\Phi^{\hat{r}} = 2 \int_{n^{\hat{z}}<0} \left(1 - \mathrm{e}^{2\tau_h / n^{\hat{z}}}\right) I_{\mathrm{irr}}(n) n^{\hat{r}} n^{\hat{z}} d\Omega \approx 2 T^{\hat{r}\hat{z}}_{\mathrm{irr}}$$

$$\Phi^{\hat{\varphi}} = 2 \int_{n^{\hat{z}}<0} \left(1 - \mathrm{e}^{2\tau_h / n^{\hat{z}}}\right) I_{\mathrm{irr}}(n) n^{\hat{\varphi}} n^{\hat{z}} d\Omega \approx 2 T^{\hat{\varphi}\hat{z}}_{\mathrm{irr}}$$

$$\phi^{\hat{z}} = \rho \left(\kappa^{\mathrm{a}} + \kappa^{\mathrm{s}}\right) \int_0^z q^{-}_{\mathrm{rad}}(z) dz$$

→ Lorentz boost to the Boyer-Lindquist coordinates, *etc...*

Vertical structure

Radiation pressure dominated disk, electron-scattering opacity:

$$\Omega_\perp^2 z = \kappa_{\rm es} \int_0^z q_{\rm rad}^- dz$$

Taking z-derivative:

$$\boxed{q_{\rm vis}^+ = \frac{\Omega_\perp^2}{\kappa_{\rm es}} = \mathrm{const}}, \quad h = \frac{\kappa_{\rm es} Q_{\rm rad}^-}{2\Omega_\perp^2}$$

...Shakura & Sunyaev (1976)

Additional assumption needed $\Rightarrow q_{\rm vis}^+ \propto \rho \Rightarrow \boxed{\rho = \mathrm{const}}$.

$\Rightarrow$ Profiles of other quantities follows.

Standard-like disk equations in Schwarzschild

Neglecting the advective and pressure terms in radial equation

Radial momentum balance (radiation force):

$$\bar{\Omega}^2 - \bar{\Omega}_{\mathrm{K}}^2 = -\alpha \dot{m} \Gamma \frac{\gamma_\varphi^3}{6x^2 \bar{\Omega}_\perp^2} \left(\frac{d\bar{\Omega}}{dx} \right)^2 t_{\mathrm{irr}}^{\bar{r}\bar{z}} y$$

Azimuthal momentum balance (drag):

$$-2\frac{d\bar{\ell}}{dx} + \frac{dy}{dx} = -\gamma_\varphi \bar{\ell} \left[2\frac{\Gamma}{\dot{m}} x t_{\mathrm{irr}}^{\bar{t}\bar{z}} - \frac{x^{1/2}}{(x-2)^{1/2}} \frac{d\bar{\Omega}}{dx} y \right]$$

Dimensionless quantities:

$$r \equiv Mx, \quad \Omega \equiv M^{-1}\Omega \quad \ell \equiv M\bar{\ell} \quad rW_\varphi^r \equiv \frac{1}{4\pi} M\dot{M} y \quad T_{\mathrm{irr}}^{\alpha\beta} = \frac{L_\infty}{4\pi M^2} t_{\mathrm{irr}}^{\alpha\beta}$$

Parameters:

$$\alpha, \boxed{\frac{\dot{M}}{L_{\mathrm{Edd}}} \equiv \dot{m},} \quad \boxed{\frac{L_\infty}{L_{\mathrm{Edd}}} \equiv \Gamma,}$$

$\Rightarrow$ There is a regime where drag dominates

Eddington vs. critical luminosity

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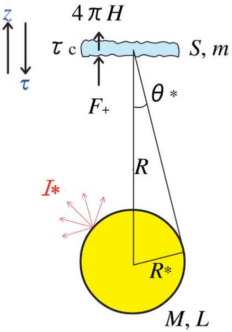


Fig. 1. Stratus above a spherical luminous source at a distance R. The quantities of the source are mass M, radius R_*, luminosity L, and surface intensity I_*; $4\pi I_* R_*^2 = L$. The quantities of the stratus are mass m, surface area S, and optical depth τ_c; $\tau_c = \kappa_{es}\Sigma = \kappa_{es} m/S$ for electron scattering opacity κ_{es}. The angle θ_* subtended by the source at the stratus distance is expressed as $\sin\theta_* = R_*/R$; $\mu_* = \cos\theta_*$. (Color online)

...Fukue 2015, PASJ 67,1

- The optical depth does matter.
- Approximate behavior of the critical Eddington factor:

$$\boxed{\Gamma_{\rm crit} \equiv \frac{L_{\rm crit}}{L_{\rm Edd}} \approx \frac{1}{2}\left(1 + \mu_* + \tau\right)}$$

Radiatively-dragged Keplerian disks

When $\alpha \dot{m} \Gamma \ll 1$, but $\dot{m}/\Gamma \sim 1 \Rightarrow$ Keplerian rotation, $\ell = \ell_{\rm K}$.

Azimuthal equation becomes

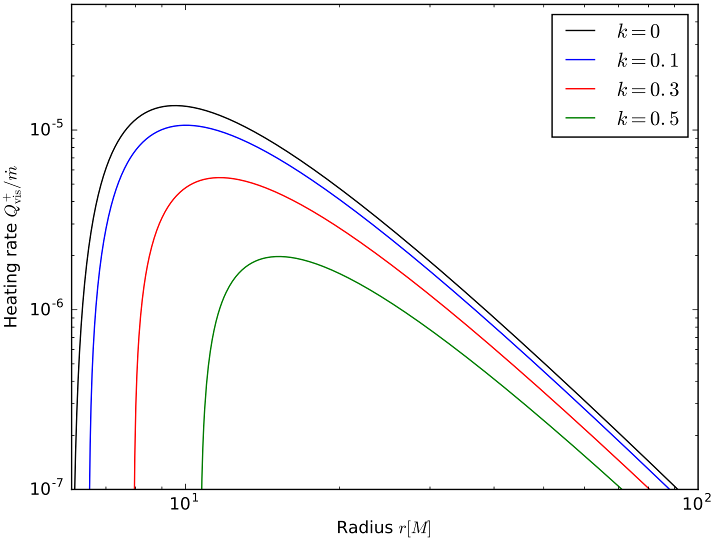
$$\frac{dy}{dx} + \frac{3y}{2x(x-3)} = \frac{x-6}{(x-3)^{3/2}} - \frac{k}{(x-3)x^{1/2}}, \quad k \equiv \frac{4\Gamma x_*^{3/2}}{3\pi \dot{m}(x_* - 2)^{1/2}}$$

It even has the analytic solution

$$y(x) = 2\sqrt{\frac{x}{x-3}}\left[\sqrt{x} + \sqrt{3}\,\mathrm{ArcTanh}\sqrt{\frac{x}{3}} - \frac{k}{\sqrt{3}}\mathrm{ArcTan}\sqrt{\frac{x-3}{3}} - C(k)\right]$$

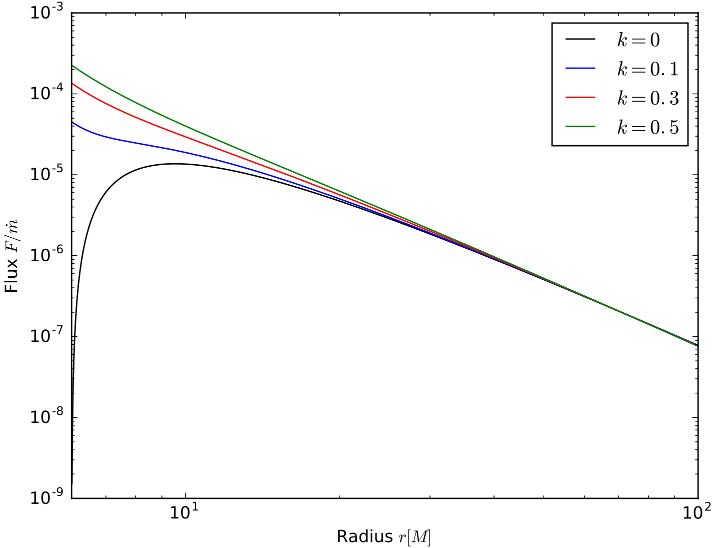
→ The integration constant $C(k)$ depends on the inner BC.
→ We use $C(k)$ corresponding to vanishing viscous torque at $r_{\rm ms}$.
→ Proper way would be to construct transonic solutions.

Viscous heating rate



→ Disks are less heated by viscosity

Local flux



→ Spectral hardening

Conclusions

Work in progress...